Effects of Initial States on Optimal Thinning and Rotation

Tianjian Cao, Metla
Kari Hyytiäinen, Metla
Olli Tahvonen, Metla
Lauri Valsta, University of Helsinki

2004 SSFE, May 13, 2004
Järvenpää, Finland

Outline

- Objectives
  - Optimal thinning patterns; Optimal rotation period; Forested land value.
- Materials and Methods
  - Norway spruce MT and OMT sites (12 plots), H(100) = 22.3 – 30.7 m; Roadside prices: sawlog 43.2 €/m³, pulpwood 29.4 €/m³ (Hyytiäinen and Tahvonen, 2002); Logging cost model (Kuitto et al. 1994).
  - Individual-tree growth model (Siitonen et al. 1996, Hynynen et al. 2002); Hooke and Jeeves’ direct search (1961); Stand Management Assistant (Valsta and Linkosalo, 1996).
- Results
- Discussion
The Optimization Problem

\[
\max_{u, u = 1, \ldots, k, \mathbf{Z}_u} \mathbb{V} = \frac{\sum_{u=1}^{k} \left[ \sum_{j=1}^{2} \frac{p_j g_j(\mathbf{Z}_{t_u}, \mathbf{h}_u)}{1 - (1 + r)^{-h}} \right] - c_u(\mathbf{Z}_{t_u}, \mathbf{h}_u)}{1 - (1 + r)^{-h}} - c_0(\mathbf{Z}_0)
\]

subject to

\[
\mathbf{Z}_{t+1} = f(\mathbf{Z}_{t}, \mathbf{t_{t+1}}), \quad u = 0, \ldots, k - 1,
\]

\[
\mathbf{t}_u \leq \mathbf{t}_{u+1} + 1, \quad u = 1, \ldots, k - 1,
\]

\[
\mathbf{h}_u \in \sigma_{t_u}, \quad u = 1, \ldots, k.
\]

\[
\max_{\mathbf{h}_u, \mathbf{t}_u, u = 1, \ldots, k} \mathbb{V}_f = \sum_{u=1}^{k} \left[ \sum_{j=1}^{2} \frac{p_j g_j(\mathbf{Z}_{t_u}, \mathbf{h}_u)}{1 - (1 + r)^{t_u-h}} \right] - c_u(\mathbf{Z}_{t_u}, \mathbf{h}_u) \frac{1}{1 + r^{t_u-h}} + \frac{\mathbb{V}^*}{1 + r^{t_u-h}}
\]

**Figure 1.** Stand development of an even-aged stand over two sequential rotations.

Finnish Forest Research Institute
**Figure 2a.** Dominant height at time of first thinning for 12 sample plots.

*Finnish Forest Research Institute*

---

**Figure 2b.** Basal area development as a function of dominant height for sparse stand (plot 4) and moderate density stand (plot 8), 3% rate of interest.

*Finnish Forest Research Institute*
Figure 2c. Dominant height at the time of first thinning as a function of number of trees per hectare, 3% rate of interest.

Finnish Forest Research Institute

Figure 3b. Basal area development as a function of stand age with 2, 4 and 6 optimized variables for each thinning at 3% rate of interest for plot 6. (var. denotes thinning variables)

Finnish Forest Research Institute
Figure 3c. Basal area before (black) and after (white) thinnings as a function of dominant height in MT sites at 1%, 3% and 5% rates of interest, and comparison with recommended before thinning level (dash line) and after thinning range (solid lines).

Figure 5a. Optimal rotation period and average diameter at rotation age in MT sites.
Figure 6a. Diameter frequency distributions at the end of rotation at 3% rate of interest for the sparse stand (plot 1) and the dense stand (plot 12), fit to 4-parameter Weibull distributions.

Finnish Forest Research Institute

Figure 6b. Diameter frequency distributions at the end of rotation at 3% rate of interest for the poor site (plot 5) and the rich site (plot 10), fit to 4-parameter Weibull distributions.

Finnish Forest Research Institute
Table 1. Maximized bare land value (pre-tax) at 1-5% rates of interest for 12 sample Norway spruce stands (€/ha)

<table>
<thead>
<tr>
<th>Interest rate</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plot 1</td>
<td>21839</td>
<td>6640</td>
<td>2859</td>
<td>1196</td>
<td>403</td>
</tr>
<tr>
<td>Plot 2</td>
<td>21839</td>
<td>6640</td>
<td>2859</td>
<td>1196</td>
<td>403</td>
</tr>
<tr>
<td>Plot 3</td>
<td>10895</td>
<td>2549</td>
<td>1389</td>
<td>70</td>
<td>292</td>
</tr>
<tr>
<td>Plot 4</td>
<td>18224</td>
<td>5652</td>
<td>2208</td>
<td>832</td>
<td>166</td>
</tr>
<tr>
<td>Plot 5</td>
<td>24280</td>
<td>7864</td>
<td>3310</td>
<td>1453</td>
<td>563</td>
</tr>
<tr>
<td>Plot 6</td>
<td>9962</td>
<td>2817</td>
<td>723</td>
<td>9</td>
<td>292</td>
</tr>
<tr>
<td>Plot 7</td>
<td>14033</td>
<td>4157</td>
<td>1456</td>
<td>418</td>
<td>366</td>
</tr>
<tr>
<td>Plot 8</td>
<td>13342</td>
<td>3911</td>
<td>1338</td>
<td>342</td>
<td>125</td>
</tr>
<tr>
<td>Plot 9</td>
<td>13681</td>
<td>4715</td>
<td>1741</td>
<td>503</td>
<td>11</td>
</tr>
<tr>
<td>Plot 10</td>
<td>20061</td>
<td>6327</td>
<td>2550</td>
<td>1034</td>
<td>286</td>
</tr>
<tr>
<td>Plot 11</td>
<td>24075</td>
<td>7781</td>
<td>2264</td>
<td>1425</td>
<td>336</td>
</tr>
<tr>
<td>Plot 12</td>
<td>12632</td>
<td>3557</td>
<td>1170</td>
<td>230</td>
<td>181</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>17222</td>
<td>5288</td>
<td>2029</td>
<td>730</td>
<td>116</td>
</tr>
</tbody>
</table>

Finnish Forest Research Institute

---

Table 5. Maximized forested land value (in parentheses) and forested land value (pre-tax) under recommended harvests (€/ha)

<table>
<thead>
<tr>
<th>Rate of interest</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plot 1</td>
<td>16213 (26318)</td>
<td>7864 (10628)</td>
<td>4842 (6033)</td>
<td>3266 (3934)</td>
<td>2315 (2877)</td>
</tr>
<tr>
<td>Plot 2</td>
<td>17108 (30202)</td>
<td>5052 (15701)</td>
<td>6142 (8477)</td>
<td>4573 (6054)</td>
<td>3573 (6469)</td>
</tr>
<tr>
<td>Plot 3</td>
<td>16396 (23050)</td>
<td>9620 (11742)</td>
<td>6474 (8049)</td>
<td>5524 (6338)</td>
<td>4545 (5321)</td>
</tr>
<tr>
<td>Plot 4</td>
<td>19133 (28377)</td>
<td>8769 (12430)</td>
<td>6170 (7370)</td>
<td>4223 (5016)</td>
<td>3040 (3677)</td>
</tr>
<tr>
<td>Plot 5</td>
<td>20715 (33528)</td>
<td>11563 (13772)</td>
<td>8101 (10014)</td>
<td>6177 (7445)</td>
<td>4932 (5926)</td>
</tr>
<tr>
<td>Plot 6</td>
<td>13243 (20896)</td>
<td>7398 (10246)</td>
<td>4306 (6730)</td>
<td>3223 (3034)</td>
<td>2304 (4122)</td>
</tr>
<tr>
<td>Plot 7</td>
<td>17725 (26584)</td>
<td>9984 (12948)</td>
<td>6860 (8391)</td>
<td>5100 (6264)</td>
<td>4005 (5064)</td>
</tr>
<tr>
<td>Plot 8</td>
<td>17500 (25782)</td>
<td>10085 (12859)</td>
<td>7105 (9558)</td>
<td>5407 (6849)</td>
<td>4550 (5404)</td>
</tr>
<tr>
<td>Plot 9</td>
<td>13666 (21630)</td>
<td>7211 (9701)</td>
<td>4779 (5950)</td>
<td>3480 (4272)</td>
<td>2704 (3332)</td>
</tr>
<tr>
<td>Plot 10</td>
<td>17811 (28546)</td>
<td>9799 (15194)</td>
<td>6746 (8342)</td>
<td>5068 (6103)</td>
<td>4015 (4770)</td>
</tr>
<tr>
<td>Plot 11</td>
<td>2064 (34444)</td>
<td>1101 (14062)</td>
<td>7440 (9316)</td>
<td>5523 (6620)</td>
<td>4331 (5183)</td>
</tr>
<tr>
<td>Plot 12</td>
<td>16051 (23910)</td>
<td>8944 (11705)</td>
<td>6122 (7641)</td>
<td>4565 (5707)</td>
<td>3612 (4635)</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>17171 (26768)</td>
<td>9338 (12466)</td>
<td>6258 (7891)</td>
<td>4727 (5775)</td>
<td>3702 (4297)</td>
</tr>
</tbody>
</table>

Finnish Forest Research Institute
Discussion

- Thinning frequency: 2-3 thin. vs. 5-6 thin.
- Timing of 1st thinning: late vs. early
- Thinning intensity: high BA vs. low BA
- Thinning type: below vs. above
- Optimal rotation: high interest rate + heavy thinning