SIZE-CLASS MODEL BASED ON SIMULATED GROWTH DATA –

Estimation and technical analysis

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BACKGROUND

- Size/stage-class model is a general representation of population dynamics

- In forestry a size-class model allows for:
  - Uneven-aged stand management
  - Optimization of thinning regime of a stand

- Unlike with an age-class model the choice of the categories is not trivial/insignificant

BACKGROUND(2)

- The classification can be based on many variables and their combinations
  - Variables include: diameter, basal area, volume, stage of development etc.
  - Transition matrix describes dynamics between adjacent stages/classes
- In forest models, size classes are typically represented by categories for saplings and diameter classes for larger trees
  - However, diameter-based size classes are of equal width

LITERATURE

- Matrix models:
  - Leslie (1945), Lefkovitch (1965), Usher (1966, 1969), Getz and Height (1989)
- Choice of the size-classes:
  - Vandermeer (1978), Moloney (1986)
- Applications in forest economics:
  - Buongiorno & Michie (1980)
  - Solberg & Haight (1991)
  - Bollandsås et al. (2008), Tahvonen (2009)

Linear model structure

- Usher's population model (1966) based on the stage-class model by Lefkovitch (1965)

$$\boldsymbol{x}_{t+1} = G\boldsymbol{x}_t$$

$$G \doteq \begin{bmatrix} b_1 + c_1 & c_2 & c_3 & \cdots & c_{N-1} & c_N \\ a_1 & b_2 & 0 & \cdots & 0 & 0 \\ 0 & a_2 & b_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_{N-1} & 0 \\ 0 & 0 & 0 & \cdots & a_{N-1} & b_N \end{bmatrix}$$

$$a_s + b_s \leq 1$$

$$a_N = 0$$

Our model: evolution specification (1)

$$\boldsymbol{n}_{t+1} = G_t(\boldsymbol{n}_t - \boldsymbol{h}_t)(\boldsymbol{n}_t - \boldsymbol{h}_t)$$

$$G(\boldsymbol{n}_t - \boldsymbol{h}_t) \doteq \begin{bmatrix} b_1(\boldsymbol{n}_t - \boldsymbol{h}_t) & 0 & 0 & \cdots & 0 & 0 \\ a_1(\boldsymbol{n}_t - \boldsymbol{h}_t) & b_2(\boldsymbol{n}_t - \boldsymbol{h}_t) & 0 & \cdots & 0 & 0 \\ 0 & a_2(\boldsymbol{n}_t - \boldsymbol{h}_t) & b_3(\boldsymbol{n}_t - \boldsymbol{h}_t) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_{N-1}(\boldsymbol{n}_t - \boldsymbol{h}_t) & 0 \\ 0 & 0 & 0 & \cdots & a_{N-1}(\boldsymbol{n}_t - \boldsymbol{h}_t) & b_N(\boldsymbol{n}_t - \boldsymbol{h}_t) \end{bmatrix}$$

$$a_s + b_s + m_s = 1$$

$$a_s(\boldsymbol{n}_t) = (1 - m_s(\boldsymbol{n}_t))\alpha_s(\boldsymbol{n}_t)$$
$$b_s(\boldsymbol{n}_t) = (1 - m_s(\boldsymbol{n}_t))(1 - \alpha_s(\boldsymbol{n}_t))$$

- Natural regeneration and in-growth omitted

Our model: Evolution specification (2)

- Growth dynamics determined by linear parameters of the diameter distribution ($\boldsymbol{n}_t$)

1. Growth transition rate (α)
   - Basal area of larger trees
   - Total basal area

2. Natural mortality ($\boldsymbol{m}$)
   - Basal area of larger trees
   - Number of trees

Data and calculation of transition variables

- Tree and stand data from MOTTI forest simulator
- Conversion of individual tree data into size-class data
  - Two different conversion methods for growth transition rate:
  - .

Proportion estimator	Increment estimator
$\alpha_s = \frac{\sum_{j \in J} I(d_j^0 \in \Delta_s) I(d_j^1 \in \Delta_{s+1}) x_j^1}{(1 - m_s) n_s^0}$	$\alpha_s = 2 \frac{\sum_{j \in J} I(d_j^0 \in \Delta_s) x_j^0 g_j}{(\bar{d}_{s+1} - \underline{d}_s) n_s^0}$

  - Natural mortality:

$$m_{si} = \frac{n_s^0 - \sum_{j \in J} I(d_{ji}^0 \in \Delta_s) x_{ij}^1}{n_s^0}$$

Estimation

- Statistical model

$$\alpha_{si} = \alpha_s(\boldsymbol{n}_{ii}) + \varepsilon_{si}^{\alpha}$$

$$m_{si} = m_s(\boldsymbol{n}_i) + \varepsilon_{si}^{m}$$

- Logistic functional form

$$\alpha_s(\boldsymbol{n}_t) = \left[1 + \exp\left(\boldsymbol{\varphi}_s{}'\mathrm{A}_s\boldsymbol{n}_t\right)\right]^{-1}$$

$$m_s(\boldsymbol{n}_t) = \left[1 + \exp\left(\boldsymbol{\psi}_s{}'\mathrm{M}_s\boldsymbol{n}_t\right)\right]^{-1}$$

$$\mathrm{A}_s\mathbf{n}_t = \begin{bmatrix} 1 & \sum_{i>s}\frac{\pi}{4}d_i^2 n_i & \sum_{s}\frac{\pi}{4}d_s^2 n_s \end{bmatrix}'$$

$$\mathrm{M}_s\mathbf{n}_t = \begin{bmatrix} 1 & \sum_{i>s}\frac{\pi}{4}d_i^2 n_i & \sum_{s} n_s \end{bmatrix}'$$

Evaluation of the size-class models (1)

- Number of size-classes
  - Increases accuracy but
  - Makes optimization more laborous

- Conversion method
  - Proportion estimator better for young forest?
  - Increment estimator better for old forest?
  - How young / old?

- Distribution of size-classes
  - No clear a priori intuition

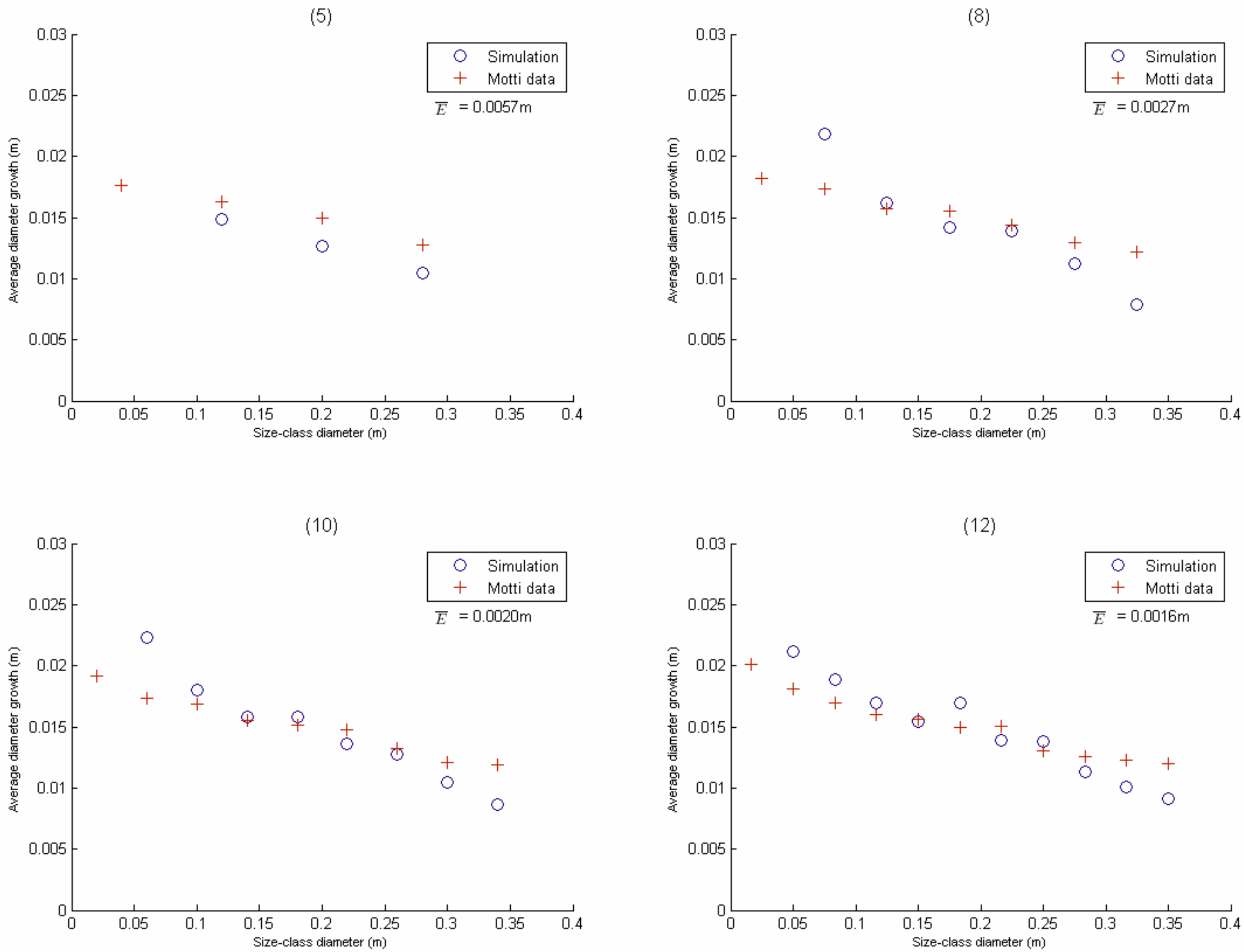
Evaluation of the size-class models (2)

- Evaluation criteria
  - Average diameter growth in size-classes
  - Diameter growth sum of size-classes
  - Basal area, volume
  - Diameter distribution

  - Optimization

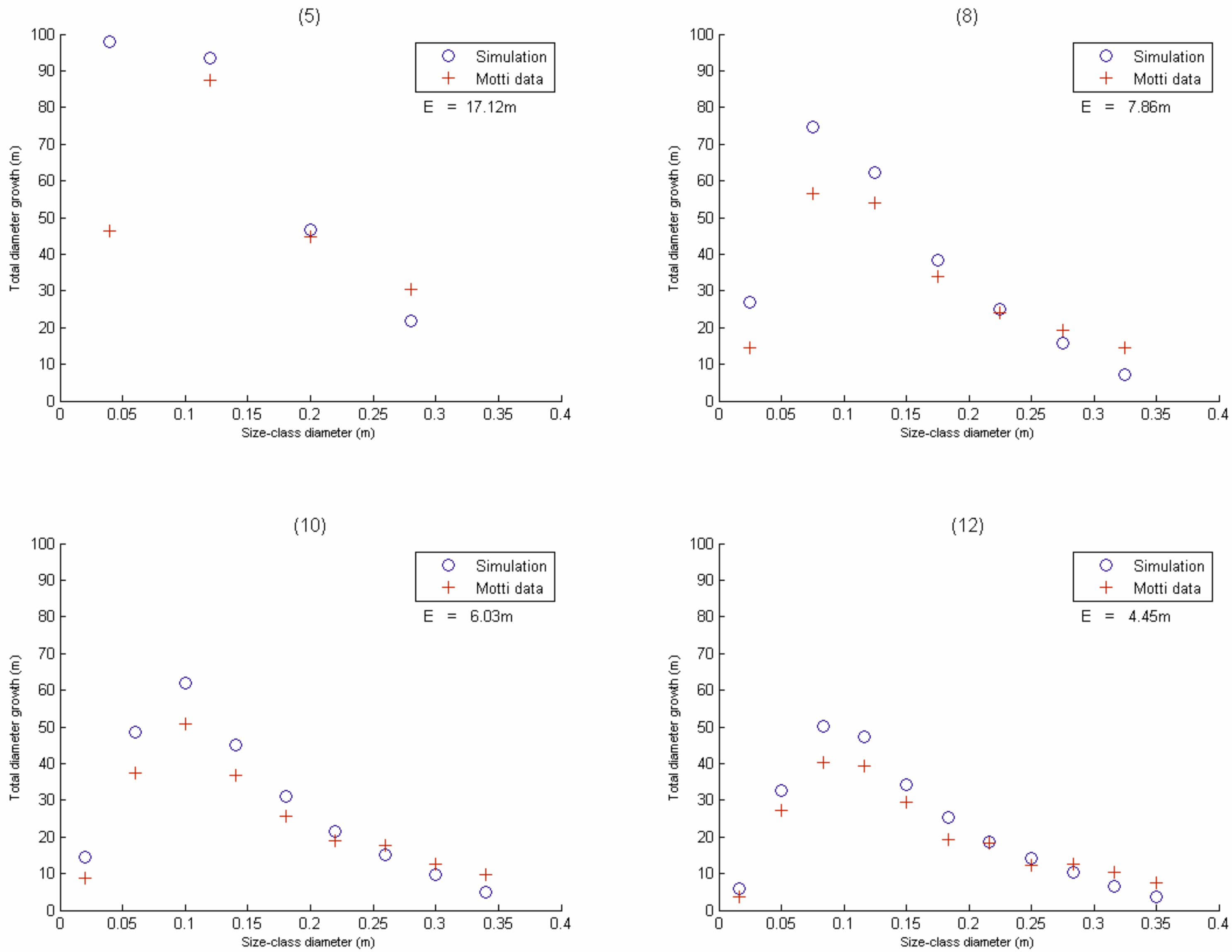
Number of size classes: Average growth

with Proportion estimator



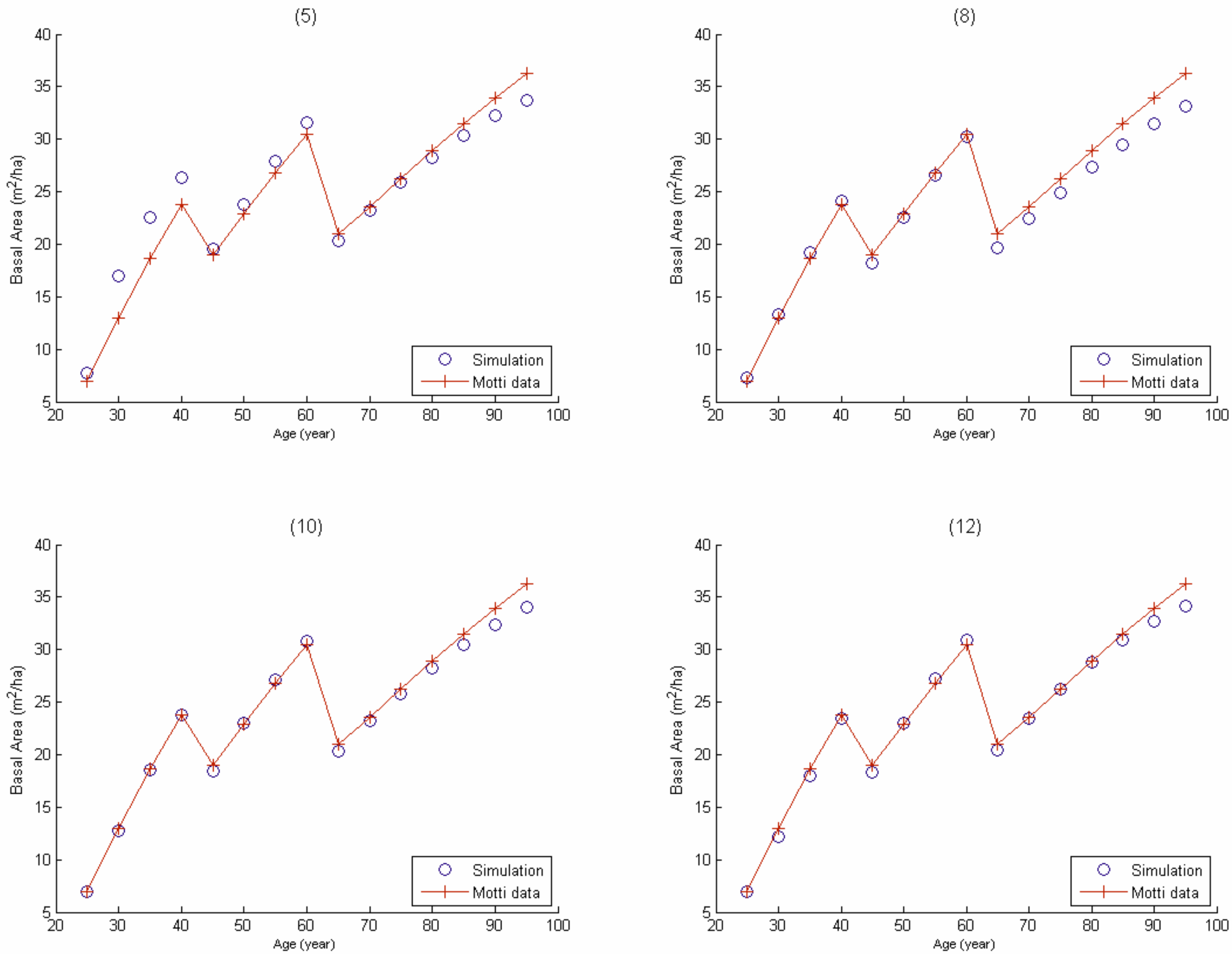
Number of size classes: Growth sum

with Proportion estimator

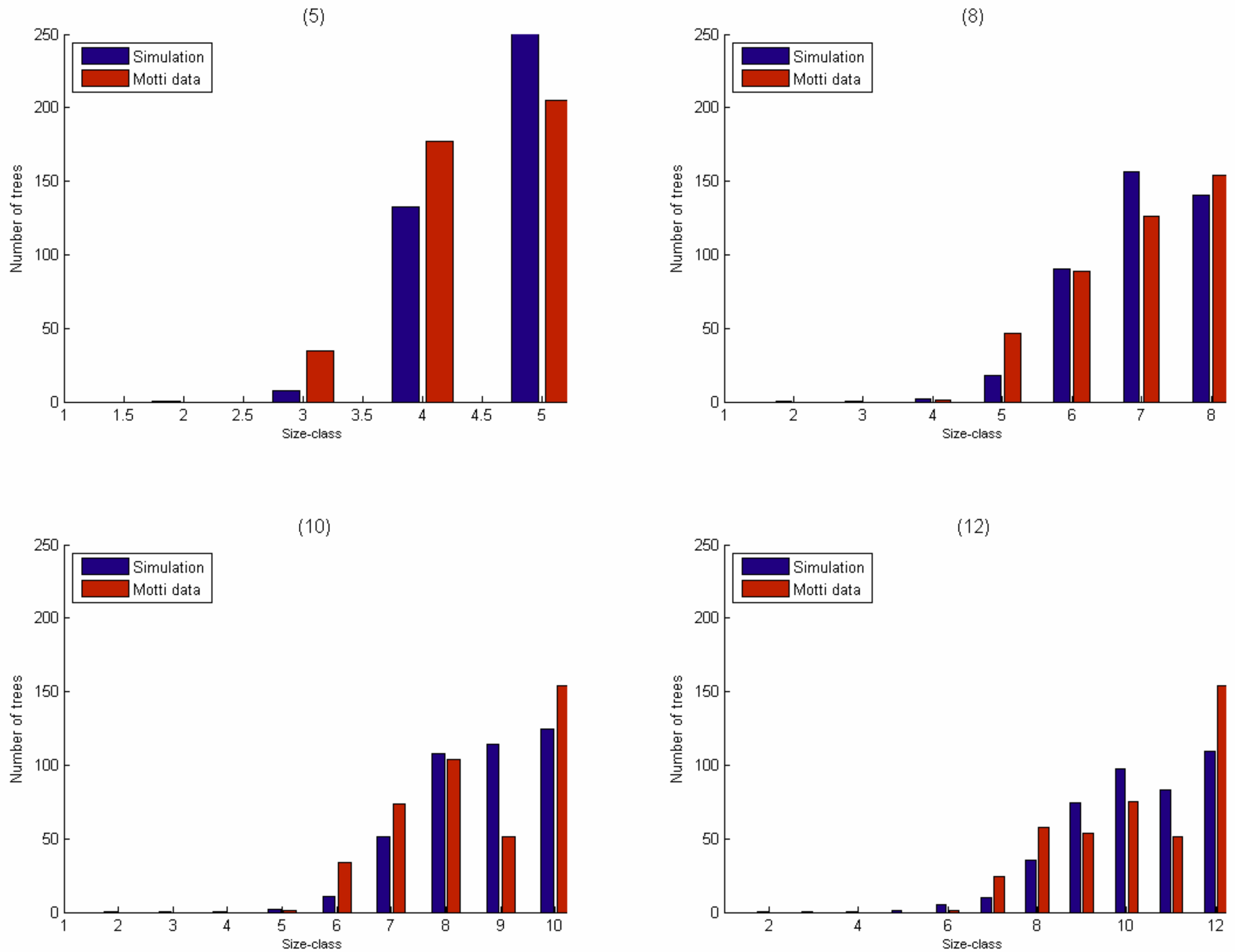


Number of size classes: Basal area

with Proportion estimator

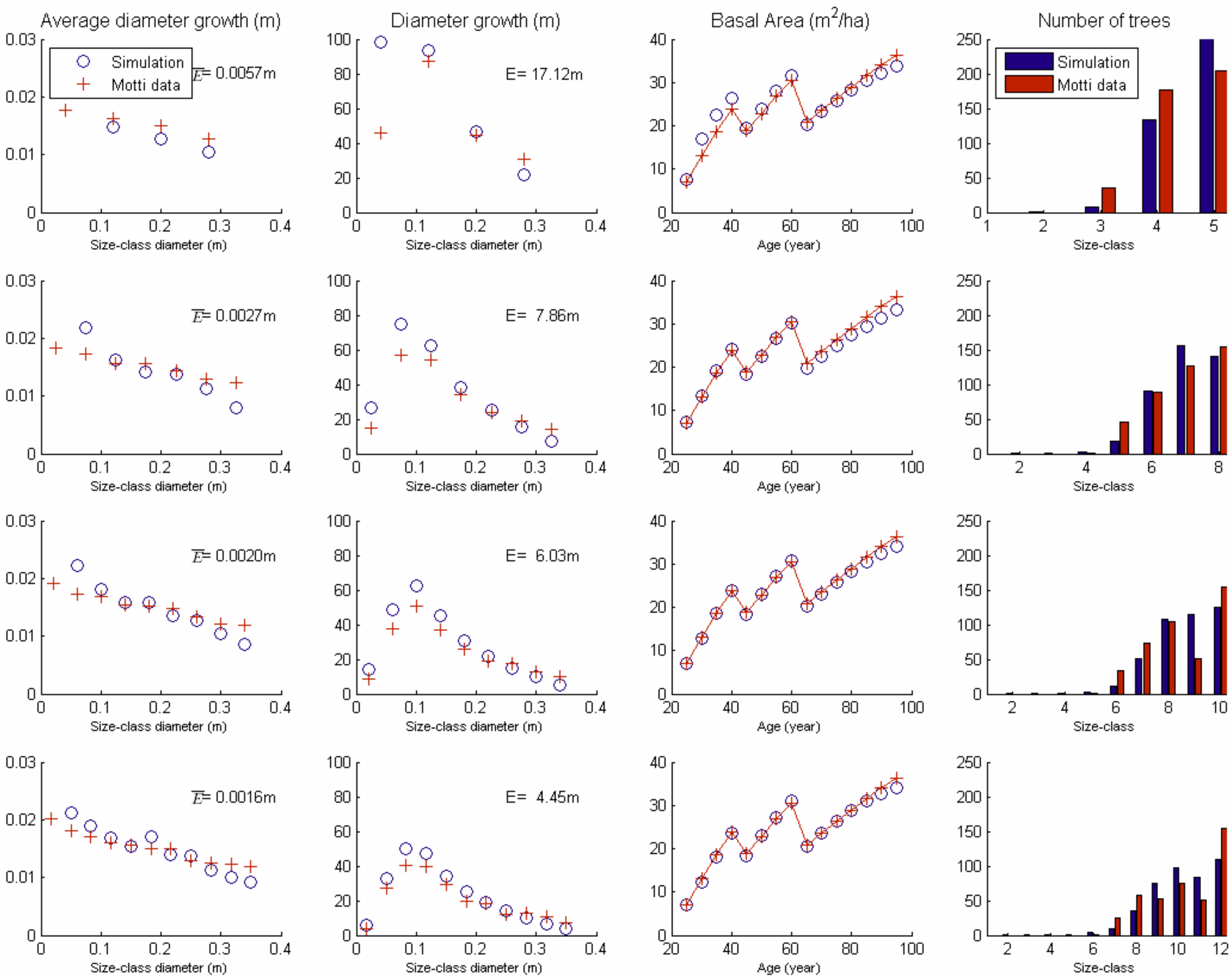


Number of size classes: Diameter distribution(t_{100}) with Proportion estimator



Number of size classes: Summary

with Proportion estimator

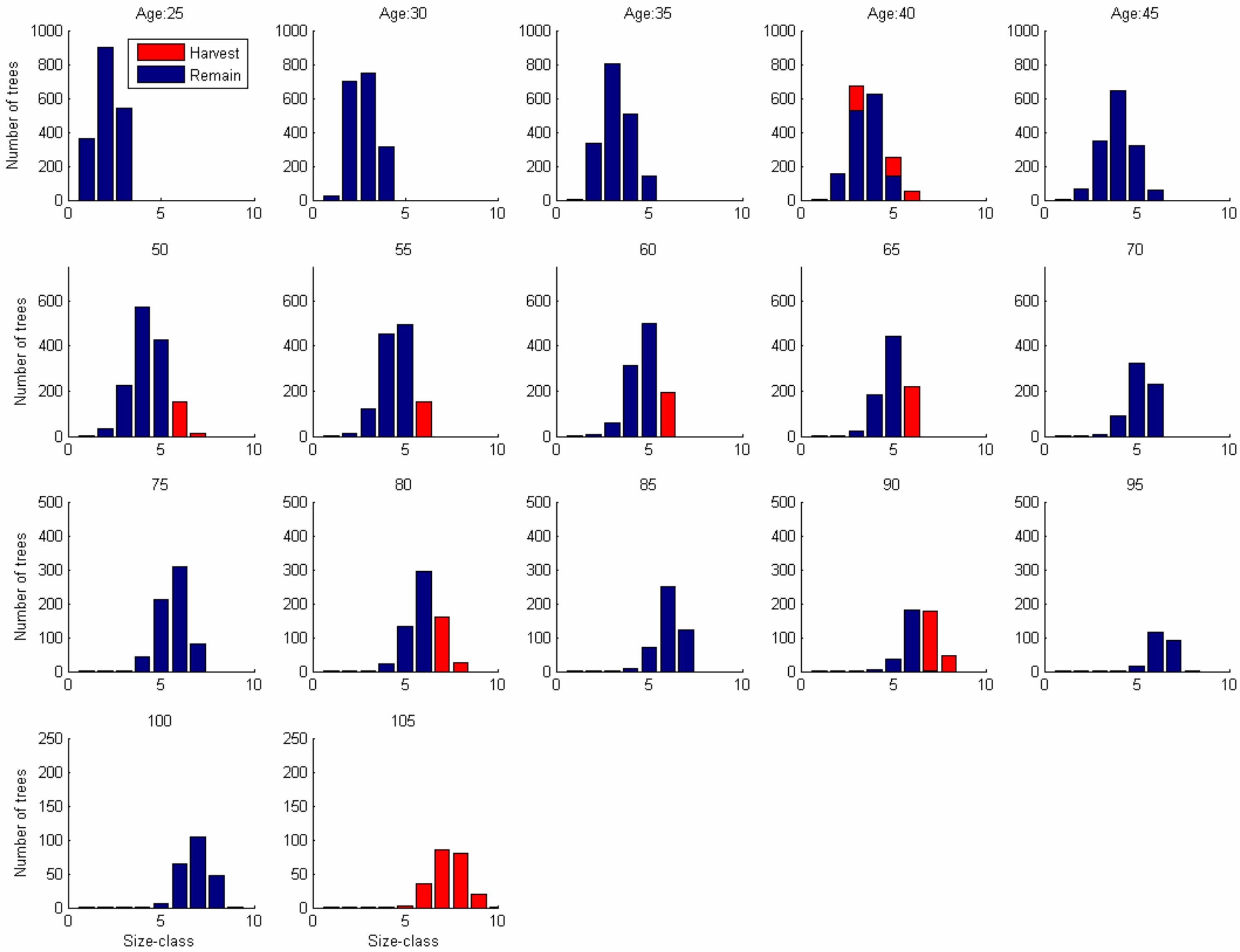


Proportion estimator: Optimization

Faustmann	5	8	10	12	10LB	10LE
T (years)	95	95	105	100	105	110
LV (euro)	3640	2960	3110	3230	2730	2950
yield (m^3/year)	6.2	6.1	6.2	6.7	6.0	6.4

MSY	5	8	10	12	10LB	10LE
T (years)	100	120	125	115	125	125
yield (m^3/year)	7.0	6.7	7.0	7.1	7.0	7.1

Proportion estimator: Optimal harvesting



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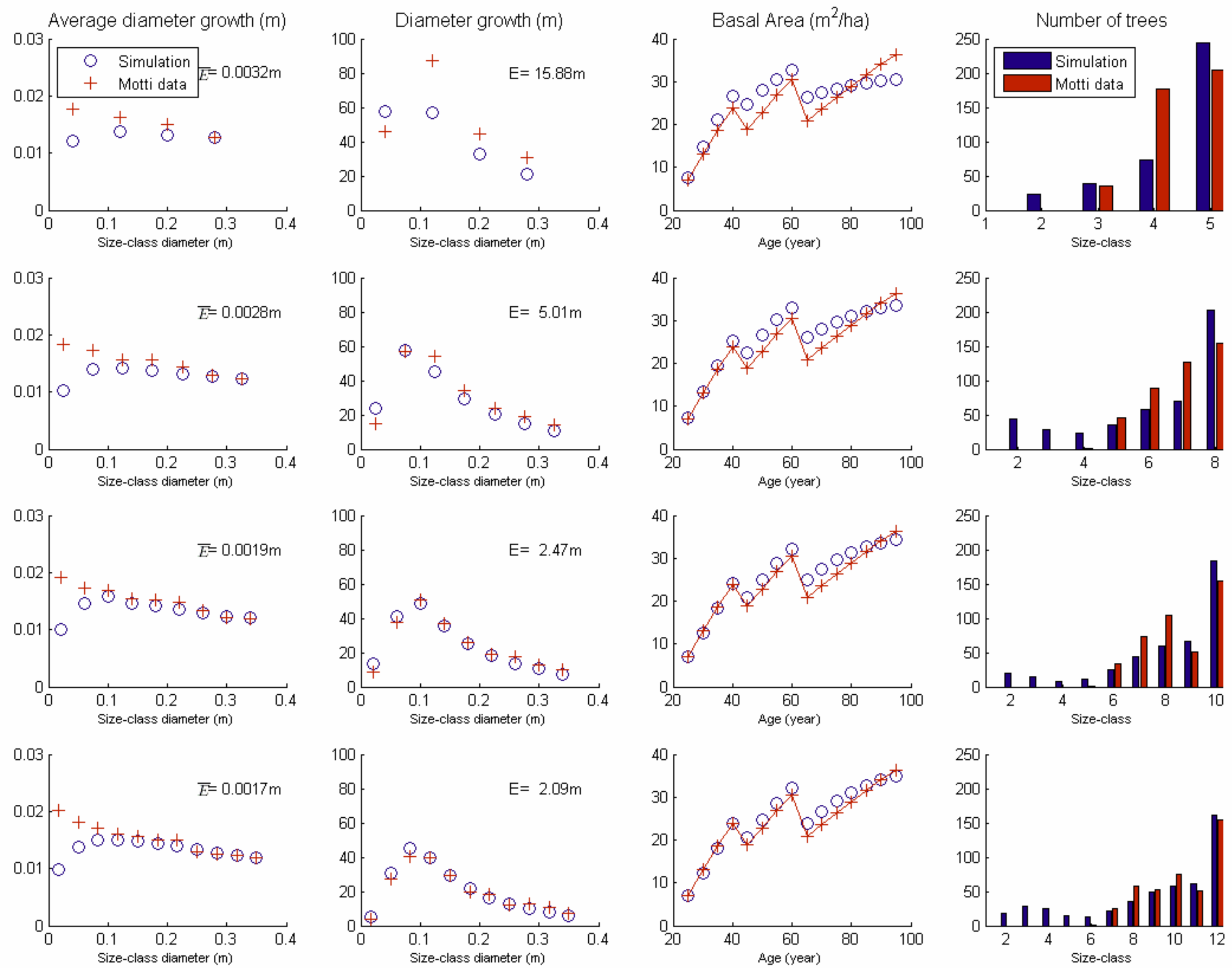
Number of size classes: Summary

with Increment estimator

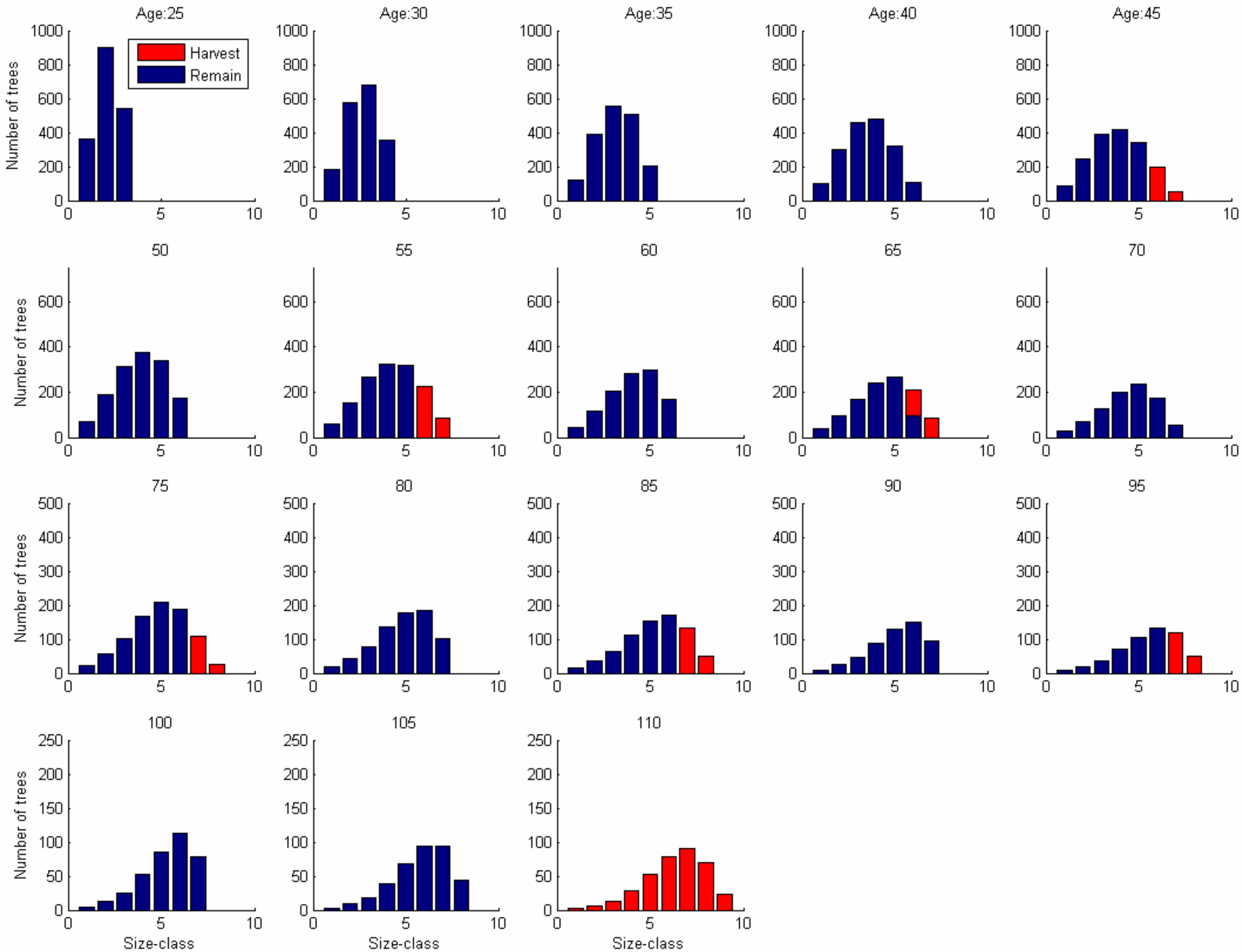


Increment estimator: Optimization

Faustmann	5	8	10	12	10LB	10LE
T (years)	120	115	110	115	110	115
LV (euro)	5410	4290	3710	3500	3790	3730
yield (m^3/year)	7.9	6.9	6.7	6.5	6.4	6.8

MSY	5	8	10	12	10LB	10LE
T (years)	115	120	125	125	120	135
yield (m^3/year)	8.4	7.7	7.4	7.3	7.1	7.6

Increment estimator: Optimal harvesting



CONCLUSIONS

- The number of classes and conversion method have an important role
- Economic indicators and accuracy of tree growth predictions both need to be evaluated when choosing a structure of a size class model
- Harvesting regimes and type can be a consequence of the model structure

Danke